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Journal of Sound and Vibration 291 (2006) 1288–1297
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Discussion

Dynamic response analysis of stiffened conoidal shells

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Received 19 March 2005; accepted 12 April 2005

Available online 18 January 2006

1. Introduction

Shell structures are commonly used as roofing units as they are capable of covering large column-free areas. These shells are often stiffened to achieve greater strength with relatively less amount of material and thus satisfy the objective of minimum weight design with economy of the material. However, they are frequently subjected to dynamic loadings in their service life and hence, the knowledge of their dynamic behaviour is important from the standpoint of analysis and design. Considerable attention has been paid to study the free vibration behaviour of isotropic stiffened cylindrical and/or elliptic paraboloid shells since the second half of the 20th century [1,2]. Mead and Bardell [3], Mustafa and Ali [4], Jiang and Olson [5], Mecitoglu and Dokmeci [6], Sinha and Mukhopadhyay [7], Stanley and Ganesan [8] and Sivasubramonian et al. [9] are some of the notable researchers investigating the free vibration behaviour of the above types of the stiffened shells. Nayak and Bandyopadhyay [10] studied the free vibration aspects of stiffened shells of five forms, i.e. cylindrical, elliptic paraboloid, hyperbolic paraboloid, hypar and conoidal shells, using finite element method. However, a few researchers including Cheng and Dade [11], Srinivasan and Krishnan [12], Jiang and Olson [13], and Sinha and Mukhopadhyay [14,15] investigated the dynamic response behaviour of isotropic stiffened cylindrical and/or elliptic paraboloid shells.

The conoidal shells being singly ruled surface are preferred to other shell forms in many places due to high aesthetic value, ease of construction and their efficiencies in permitting more natural light and hence, they are considered as industrially important structures. However, there is a relative scarcity of information for dynamic behaviour of stiffened conoidal shells, as opposed to stiffened cylindrical and elliptic paraboloid shells. While Nayak and Bandyopadhyay [10,16] only presented the free vibration analysis and design aids of stiffened conoidal shells employing the finite element method, the dynamic response analysis of these shells subjected to any dynamic loading is yet to be carried out.

The aim of this paper is to extend the finite element formulation developed by the authors for the free vibration analysis of stiffened shallow shells [10] in order to perform the dynamic response analysis. It further reports the results of displacement response of stiffened conoidal shells with parametric variation of stiffeners

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doi:10.1016/j.jsv.2005.04.035

under different uniformly distributed transient load cases. The findings of this analysis are useful for the designers in stiffening the conoidal shells efficiently subjected to dynamic loadings.

2. Finite element formulation

The nine node doubly curved isoparametric thin shallow shell element and the three node curved isoparametric beam elements are appropriately combined to form doubly curved stiffened shell element. The element stiffness and mass matrices of the shell, x-directional stiffener and y-directional stiffener elements of conoidal shells (Fig. 1) with stiffeners are derived employing the standard procedure of the finite element method, as obtained by the same authors in Ref. [10] for the free vibration analysis of stiffened shallow shells. The element stiffness $[K_e]$ and mass $[M_e]$ matrices of the stiffened shell element are then obtained by adding the element stiffness and mass matrices of the shell, x-directional stiffener and y-directional stiffener elements. The element load vector $\{P_e\}$ is derived as

$$\{P_e\} = \int\int [N]^{\mathrm{T}}\{p\}\,\mathrm{d}x\,\mathrm{d}y, \tag{1}$$

where $[N]$ is the shape function matrix of the shell element and $\{p\}$ is expressed as

$$\{p\} = [p_x \;\; p_y \;\; p_z \;\; \mu_x \;\; \mu_y]^{\mathrm{T}}, \tag{2}$$

in which p_x, p_y and p_z are the uniformly distributed loads per unit area along x, y and z axes, respectively, and μ_x and μ_y are the moments per unit area along x and y axes, respectively.

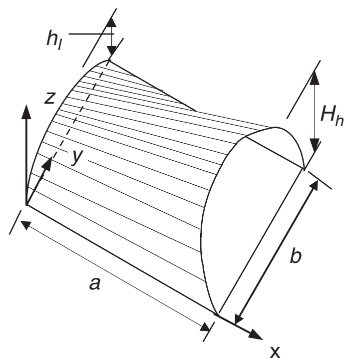
The element stiffness $[K_e]$ and mass $[M_e]$ matrices and load vector $\{P_e\}$ are then assembled to obtain the global stiffness $[K]$ and mass $[M]$ matrices and load vector $\{P\}$ of the stiffened shells, respectively. Then, the governing equation of the forced vibration of stiffened conoidal shells is obtained as

$$[M]\{\ddot{d}\} + [K]\{d\} = \{P\}, \tag{3}$$

where the load vector $\{P\}$ is transient in character and is solved using Newmark's method to get the dynamic responses.

3. Numerical results and discussions

The validity of the present formulation is first established by comparing the present results of free and forced vibration of specific problems with those available in the literature. Since, the results of dynamic responses of the stiffened conoidal shells are not available in the literature, the available results of the free vibration of conoidal shells and forced vibration of stiffened plates and stiffened cylindrical shells are



$$z = 4\left[h_l + \left(H_h - h_l\right)\frac{x}{a}\right]\left[\frac{y}{b} - \frac{y^2}{b^2}\right]$$

Fig. 1. Typical conoidal shell.

Table 1
Non-dimensional fundamental frequency for simply supported conoidal shells (Fig. 1, $ab/h^2 = 10000$, $h_l/b = 0.00$, $\nu = 0.15$ and $\varpi = \omega(ab/h)\sqrt{\rho/E}$)

Shallowness parameters	b/a	Ref. [17]	Present results
$(H_h - h_l)/b = 0.10$	0.50	10.870	10.811
$(H_h - h_l)/b = 0.20$	0.50	14.990	14.752
$(H_h - h_l)/b = 0.10$	1.0	13.659	13.590
$(H_h - h_l)/b = 0.20$	1.0	18.590	18.276

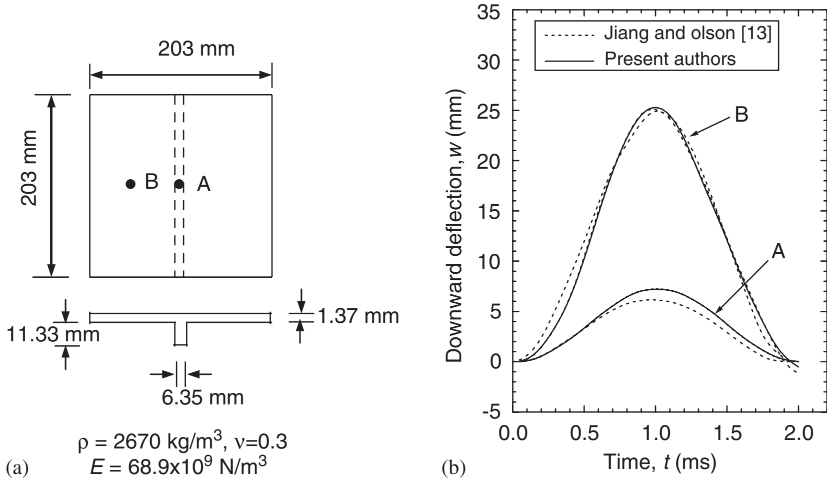


Fig. 2. Linear elastic transient response of simply supported 2-bay stiffened panel under a step load p_0 ($p_0 = 0.3$ Mpa, $\Delta t = 0.05$ ms): (a) structural configuration and material properties and (b) displacement responses.

compared to validate the formulation for the forced vibration analysis of the stiffened conoidal shells. The converged results of non-dimensional fundamental frequency [$\varpi = \omega(ab/h)\sqrt{\rho/E}$] of conoidal shells (Fig. 1) without stiffeners, obtained by the present formulation are presented in Table 1, along with those obtained by Stavridis [17]. The results from both the sources are in good agreement. The converged dynamic responses of downward deflection at two typical points A and B of simply supported 2-bay stiffened plate panel, as shown in Fig. 2(a), are obtained employing the present code and are presented in Fig. 2(b) along with those of Jiang and Olson [13]. The present results are in good agreement with the earlier ones. Transient dynamic response of a cylindrical shell with two straight edge stiffeners and supported on rigid diaphragm at the curved edges, as shown in Fig. 3(a) is carried out taking the self-weight of the shell as a step load. The time step of 7 μs is considered for the response analysis. Sinha and Mukhopadhyay [15] investigated this problem using higher order triangular shallow shell element. Fig. 3(b) shows good agreement of vertical displacement responses of Sinha and Mukhopadhyay [15] at central point C and mid-span edge point D with those of the present formulation. Hence, the present code is validated for the dynamic response analysis of the stiffened conoidal shells.

Thereafter, additional examples of clamped stiffened conoidal shells are taken up for parametric studies on the dynamic response behaviour under three different uniformly distributed transient load cases: step load of infinite duration, step load of finite duration and half sine load of finite duration, as shown in Fig. 4. In view of the fact that there are large number of variable parameters, namely number, orientation and type of stiffeners and stiffener depth to shell thickness ratio (d_{st}/h), a preliminary study is made to select each of the above-mentioned parameters for load case I (Fig. 4(a)). Thereafter, the dynamic response behaviour of the stiffened conoidal shells with selected parameters, as obtained from the preliminary study, is investigated for the three different load cases.

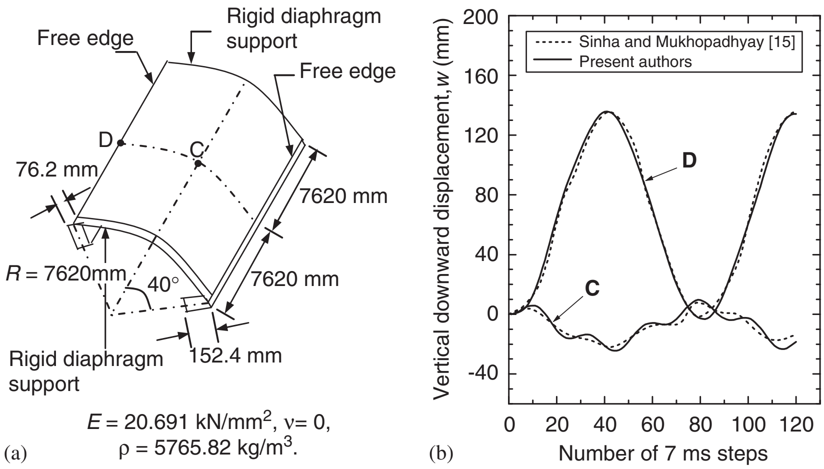


Fig. 3. Transient dynamic responses of points C and D of stiffened shell-roof structure under a dead weight step load ($\Delta t = 7\,\text{ms}$): (a) structural configuration and material properties and (b) displacement response.

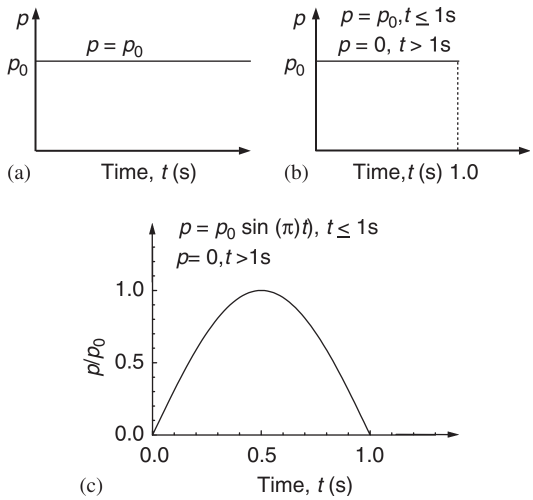


Fig. 4. Uniformly distributed load (p) vs. time (t) history. (a) Load case I: uniformly distributed step load of infinite duration, (b) load case II: uniformly distributed step load of finite duration (1 s) and (c) load case III: uniformly distributed half-sine load of finite duration (1 s).

In all the additional examples, the following parameters have been considered as constant.

$$a/b = 1, \quad b/h = 250, \quad b/H_h = 0.15, \quad h_l/H_h = 0.25, \quad b_{st}/h = 2, \quad v = 0.15,$$

$$b = 25.0\,\text{m}, \quad E = 25.491 \times 10^9\,\text{N/m}^2 \quad \text{and} \quad \rho = 2500.0\,\text{kg/m}^3,$$

where a, b and h are length (span), width (chord width) and thickness of conoidal shells, respectively, b_{st} and d_{st} are width and depth of stiffeners, v, E and ρ are Poisson's ratio, Young's modulus and mass density of the material, respectively, and H_h and h_l are defined in Fig. 1.

In the preliminary study for the load case I, the total time duration of 1 s is considered for the response analysis, taking into account the clarity of the response figures. However, the response characteristics of selected examples of stiffened conoidal shells for all the three load cases are investigated for a duration of 2 s in order to get the trend of the response behaviour beyond 1 s time at which the load cases II and III are withdrawn. The attention is mainly focused on the critical vertical displacement response only. In all figures,

Table 2
Schematic layout of the preliminary study for Load case I

Sl. No.	Variable parameter(s) to be studied	Other selected or constant parameters	Selected parameter(s)	Location of results
(a)	Number and orientation of stiffeners: number up to 10 and all three orientations (x-, y- and orthogonal)	Type of stiffeners = Ecc_t, and $d_{st}/h = 6$	5×5 orthogonal	Tables 3 and 4
(b)	Type of stiffeners: Conc, Ecc_b & Ecc_t	Number and orientation = 5×5 orthogonal*, and $d_{st}/h = 6$	Ecc_t	Fig. 5
(c)	Stiffener depth to shell thickness ratio (d_{st}/h): (d_{st}/h) up to 10	Number and orientation = 5×5 orthogonal*, and type of stiffeners = Ecc_t*	$d_{st}/h = 6$	Fig. 6

Note: *—selected parameter, Conc—concentric, Ecc_b—eccentric at bottom and Ecc_t—eccentric at top.

the critical vertical displacement (w) is expressed in non-dimensional form (w_{nd}) as

$$w_{nd} = \frac{wEh^3}{12(1-v^2)p_0b^4}, \tag{4}$$

where p_0 is the peak value of uniformly distributed transient load.

From the convergence study, the time step of 1 ms is taken for the time integration and mesh sizes of 12×6 for the half shells/12×12 for the full shells are considered in order to obtain the converged response characteristics.

3.1. Preliminary response study for selection of parameters

Several examples of stiffened conoidal shells with clamped boundary conditions are now considered for dynamic response analysis under uniformly distributed step load of infinite duration (Load case I), as shown in Fig. 4(a), with varying number of stiffeners at equal spacing for the three orientations (along x-/y-/orthogonal directions) in order to arrive at the particular combination of 5×5 orthogonal stiffeners. Further, the three types of rectangular stiffeners (concentric, eccentric at top and eccentric at bottom) of the 5×5 orthogonally clamped stiffened conoidal shells are considered to select the "eccentric at top" stiffeners.

Thereafter, a particular d_{st}/h equal to 6 is arrived at for all other parameters specified earlier. The schematic layout of the above-mentioned preliminary study for Load case I is furnished in Table 2. A brief mention of each of the three items of Table 2 is made in the following.

3.1.1. Number and orientation of stiffeners

In this section, the dynamic response analysis of the clamped square conoidal shells having rectangular stiffeners of the same size at equal spacing is investigated under the uniformly distributed step load of infinite duration (Load case I) with respect to number and orientation of stiffeners. The numbers of stiffeners provided are 1, 2, 3, 4, 5, 6 and 10 separately along the x- and y-directions and 1×1, 2×2, 3×3, 4×4 and 5×5 in orthogonal directions, keeping the type of the stiffeners constant (i.e., eccentric at top) to study the effects of the number and orientation of stiffeners.

Typical variations of non-dimensional critical vertical displacements (w_{nd}) with time for the above cases of the stiffened shells are obtained. It is worth mentioning that the maximum peak value of critical vertical displacement response for each case is the controlling factor to assess the performance of the structure from the design point of view. Hence, the maximum peak values of non-dimensional critical vertical displacement responses for all the cases considered here are presented in Table 3 in descending order to get the relative performance of the stiffened conoidal shells in the forced vibration with respect to the number and orientation of stiffeners.

Interestingly, it is observed that the maximum peak values of critical vertical displacement responses are gradually decreasing with increasing number of stiffeners in all the cases considered here and they are grouped into six sub-heads, as shown in Table 4. This table presents odd and even number of stiffeners along x-, y- and orthogonal orientations separately to reveal that the maximum peak values of displacement responses are gradually decreasing for each of the groups with the increasing number of stiffeners. The fourth group, however, has only the exception where the shell with 0×6 stiffeners shows higher maximum value of displacement response than that with 0×4 ones.

A critical study of Tables 3 and 4 reveals the following:

(i) The orthogonal orientation having odd number of stiffeners in each direction (1×1, 3×3 and 5×5) is showing monotonically decreasing peak values of the displacement responses with increasing number of

Table 3
Maximum peak values of non-dimensional critical vertical displacement responses, in descending order, of clamped conoidal shells with "eccentric at top" stiffeners under uniformly distributed step load p_0 of infinite duration

Maximum peak value of non-dimensional critical vertical displacement response $(w_{nd})_{\max} \times 10^6$	Number and orientation of stiffeners ($n_x \times n_y$)
−23.6	0×2
−23.5	0×0
−23.3	0×1
−20.0	2×0
−19.8	2×2
−18.6	4×0
−17.2	1×1
−16.4	0×3
−14.6	1×0
−14.6	6×0
−13.7	3×0
−13.7	0×5
−12.9	0×6
−12.6	0×4
−12.0	5×0
−10.2	4×4
−9.8	10×0
−9.5	3×3
−7.8	0×10
−7.5	5×5

Table 4
Relative performance of different numbers and orientations of stiffeners with respect to the response behaviour, in decreasing order, in each row

Sl. No.	Different cases	Number and orientation of stiffeners
1	Odd number of x-directional stiffeners	1×0, 3×0 and 5×0
2	Even number of x-directional stiffeners	2×0, 4×0, 6×0 and 10×0
3	Odd number of y-directional stiffeners	0×1, 0×3 and 0×5
4	Even number of y-directional stiffeners	0×2, 0×6, 0×4 and 0×10
5	Orthogonal stiffeners having odd number of stiffeners in each direction	1×1, 3×3 and 5×5
6	Orthogonal stiffeners having even number of stiffeners in each direction	2×2 and 4×4

stiffeners. The same is also the trend for even number of orthogonal stiffeners (2×2 and 4×4). However, lower peak values of displacement response for 1×1 and 3×3 orthogonal stiffeners than those for 2×2 and 4×4 stiffeners, respectively, establish the superiority of orthogonal orientation with odd number of stiffeners in each orthogonal direction (Table 3 and Sl. Nos. 5 and 6 of Table 4).

(ii) Odd number of stiffeners either along *x*- or *y*-direction gradually reduces the peak value of the displacement response with the increase in number of stiffeners (Sl. Nos. 1 and 3 of Table 4).

(iii) Odd number of stiffeners along *x*-direction is found to be superior to the same along *y*-direction as it shows comparatively lower peak values of the displacement response (Table 3).

(iv) Even number of stiffeners along *x*-direction gradually reduces the peak value of the displacement response with the increase of the number of stiffeners (Sl. No. 2 of Table 4).

(v) Even number of stiffeners along *y*-direction shows in general a reduction of the peak value of the displacement response with the increase of the number of stiffeners, with the exception of 4 number of stiffeners showing less peak value of the displacement response than 6 number of stiffeners along the same orientation (Sl. No. 4 of Table 4).

(vi) Even number of stiffeners along *y*-direction is found to be better than that along *x*-direction except for the 2 number of stiffeners along *y*-direction, which interestingly exhibits marginally higher peak value of the displacement response than the same when there is no stiffener (Table 3).

(vii) The relative comparison of the orthogonal orientation with odd number of stiffeners (1×1, 3×3 and 5×5) in each direction with the other two orientations (*x*-/*y*-) having the same number of stiffeners (2, 6 and 10, respectively) reveals the superiority of the orthogonal orientation (Table 3).

(viii) On the other hand, orthogonal orientation with 2 numbers of stiffeners in each direction is found to be inferior to 4 numbers of stiffeners along either *x*- or *y*-direction. However, the *y*- orientation (with 0×4) shows the best performance (Table 3).

The above detailed study of number and orientation of stiffeners clearly indicates the superiority of orthogonal orientation with 5 numbers of stiffeners along each direction. Accordingly, 5×5 orthogonal stiffeners are considered for the problem of the subsequent sections.

3.1.2. Types of stiffeners

The response characteristics of the clamped conoidal shells with 5×5 orthogonal stiffeners are studied for Load case I with respect to the three types of stiffeners: concentric, eccentric at bottom and eccentric at top. The variation of non-dimensional critical vertical displacement (w_{nd}) with time is presented in Fig. 5. Though a quick glance of the response characteristics of this example shows the superiority of concentric stiffeners followed by eccentric at top and eccentric at bottom in the order of preference, a critical inspection of the

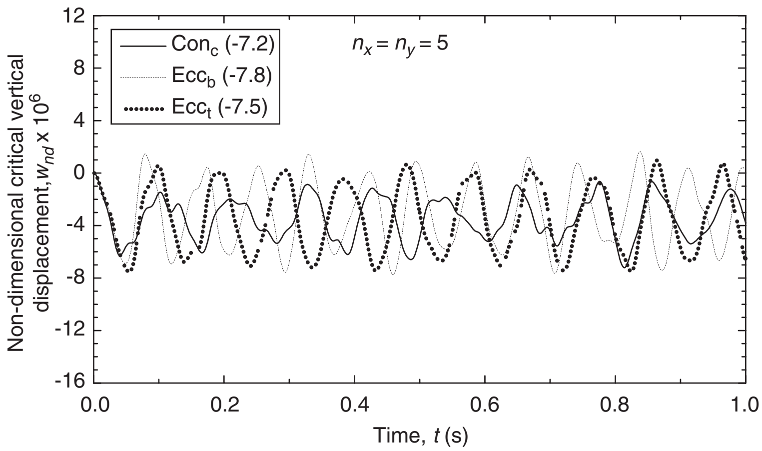


Fig. 5. Dynamic response of clamped conoidal shell stiffened with orthogonal stiffeners of different types under uniformly distributed step load p_0 of infinite duration (Conc: concentric, Ecc_b: eccentric at bottom, Ecc_t: eccentric at top).

nature of the response curves reveals the similarity of their trend. Moreover, the peak values as mentioned in the inset legends of Fig. 5 are also close to each other. It is worth mentioning that the conoidal shells exhibit poor performance with concentric type of 5×5 orthogonal stiffeners than those with other two types while performing the free vibration analysis as reported in the literature [10].

The "eccentric at top" type of the stiffeners, however, has been selected because of its superiority in the free vibration study [10]. Moreover, its performance is close to that of the most preferred concentric stiffeners in the forced vibration study.

3.1.3. Stiffener depth to shell thickness ratio (d_{st}/h)

In this section, the dynamic response analysis of clamped conoidal shells with 5×5 orthogonal "eccentric at top" stiffeners is investigated under Load case I for the increasing values of d_{st}/h up to 10 (Fig. 6). The discrete values of d_{st}/h considered are 0, 2, 4, 6, 8 and 10. The maximum peak value in each case is shown within brackets in the respective inset legend in Figs. 6(a) and (b). It is observed that the maximum peak value of the response of w_{nd} decreases significantly when the value of d_{st}/h increases from 0 to 6. Further increase of the (d_{st}/h) ratio hardly decreases the maximum peak value of the response of w_{nd} as shown in Fig. 6(b).

The above study and several trials of the forced vibration analysis with a few other d_{st}/h values, though not presented here, reveal that the optimum benefit can be achieved for the stiffened conoidal shells with (d_{st}/h) ratio equal to 6. Accordingly, the authors select (d_{st}/h) ratio equal to 6 for further parametric study. It is worth mentioning that the authors have adopted (d_{st}/h) ratio as 6 for all the earlier cases.

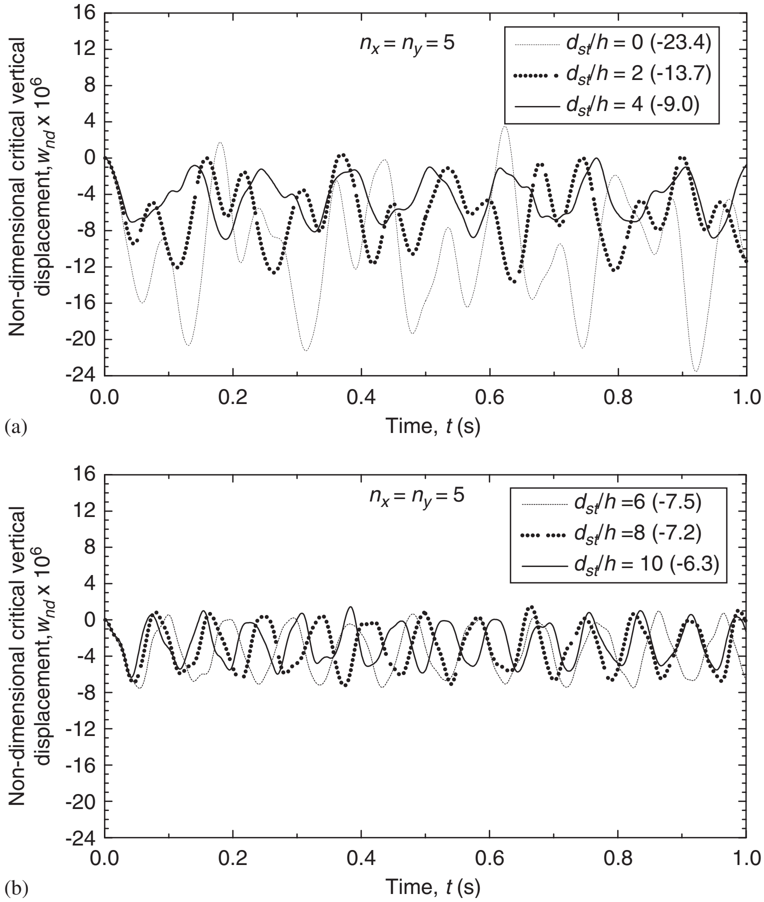


Fig. 6. Dynamic response of clamped "eccentric at top" orthogonally stiffened conoidal shell with different stiffener depth to shell thickness ratio under uniformly distributed step load p_0 of infinite duration: (a) $d_{st}/h = 0$, 2 and 4, and (b) $d_{st}/h = 6$, 8 and 10.

3.2. Response study of stiffened conoidal shells with the selected parameters under the three transient load cases

The variation of non-dimensional critical vertical displacements (w_{nd}) with time is presented in Fig. 7 for stiffened conoidal shells subjected to the three transient load cases (Fig. 4), along with the maximum displacement for static load equal to the peak value of the transient loads. The maximum peak values in each of the load cases are shown within brackets in the respective inset legend of Fig. 7. The magnification factors (i.e., ratio of the maximum peak value of critical transient displacement response to the maximum static displacement) of all the transient load cases for the shells are presented in Table 5.

The non-dimensional critical vertical displacements (w_{nd}) of the stiffened conoidal shells (Fig. 7) are seen to oscillate at different points of time for all three transient loads, resulting even upward displacement of magnitude exceeding the static ones. This observation brings out the importance of the dynamic analysis because the static simplification of a dynamic problem using a suitable magnification factor cannot account for the reversal of different shell actions.

As seen from Fig. 7, the effect of the step loads (Load cases I and II) on the critical vertical displacement is more pronounced than that of the half sine load (Load case III), yielding higher values of the magnification factor (Table 5). Moreover, the response curves for Load cases I and II show a greater number of maxima and minima. Hence, Load cases I and II are more severe than Load case III. This is due to the fact that the sine load is gradually applied and withdrawn, unlike step loads, and the impulse of the sine load is much less compared to step loads, although the peak values of all the three loads are identical. For the sine load, the critical displacement response exhibits a variation with constant amplitude of values changing harmonically with a time period almost equal to the fundamental time period when the load is withdrawn. Since the load is gradually withdrawn, the structure exhibits free vibration and transient effect seems to disappear. These are, however, absent for Load case II when the load is suddenly withdrawn. The magnification factors of the critical vertical displacements are almost equal for Load cases I and II. This means that the transient effects of Load case I decay after some time and the dynamic analysis may be carried out up to that time only.

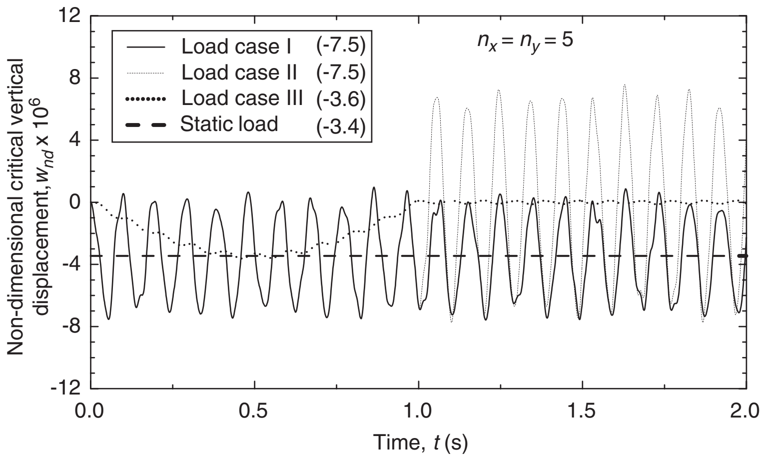


Fig. 7. Dynamic response of clamped "eccentric at top" orthogonally stiffened conoidal shell under different load cases.

Table 5
Magnification factors of clamped "eccentric at top" orthogonally stiffened conoidal shells for the three transient load cases

Load case	Magnification factor
I	2.21
II	2.21
III	1.06

4. Concluding remarks

The brief review has shown the lack of information of the dynamic behaviour of the stiffened conoidal shells subjected to transient loads. The earlier finite element code developed by the authors [10] has been extended to perform the dynamic response analysis of stiffened conoidal shells with sufficient accuracy. The present parametric study of the clamped stiffened conoidal shells subjected to uniformly distributed transient load (Load case I) reveals the characteristic behaviour and intrinsic features of displacement response which are useful in selecting the number, orientation and type of stiffeners and stiffener depth to shell thickness ratio. Moreover, from the dynamic response analysis of stiffened conoidal shells subjected to the three different transient load cases considered here, it is observed that the step loads (Load cases I and II) are more severe than the sine load (Load case III) as the sine load is applied and withdrawn gradually.

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